Please check the examination details below before entering your candidate information

Candidate surname	Other names
MODEL SOLUTIONS	

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Monday 24 June 2024

Afternoon (Time: 1 hour 30 minutes) | Paper reference **9FM0/4A**

Further Mathematics

Advanced

PAPER 4A: Further Pure Mathematics 2

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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Pearson

1. **In this question you must show detailed reasoning.**

Use Fermat's Little Theorem to determine the least positive residue of

$$21^{80} \pmod{23}$$

(4)

Recall Fermat's Little Theorem:

If p is prime and a is an integer not divisible by p, then

$$a^{p-1} \equiv 1 \pmod{p}$$

$$21^{23-1} \equiv 1 \pmod{23} \quad 1$$

Since $80 = 22 \times 3 + 14$,

$$21^{80} = (21^{22})^3 \times 21^{14} \quad 1$$

$$\Rightarrow (21^{22})^3 \equiv 1^3 \equiv 1 \pmod{23}$$

Now compute $21 \bmod 23$ to calculate $21^{14} \bmod 23$.

$$21 \equiv -2 \pmod{23}$$

$$\Rightarrow 21^{14} \equiv (-2)^{14} \pmod{23}$$

$$(-2)^{14} = 2^{14} = (2^7)^2$$

$$2^7 = 128 \equiv 13 \pmod{23} \quad \text{as } 128 \div 23 = 5 \text{ r } 13$$

$$13^2 = 169 \equiv 8 \pmod{23} \quad 1 \quad \text{as } 169 \div 23 = 7 \text{ r } 8$$

So $21^{80} \equiv 8 \pmod{23}$ 1

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2. Determine a closed form for the recurrence system

$$u_1 = 4 \qquad u_2 = 6$$
$$u_{n+2} = 6u_{n+1} - 9u_n \qquad (n = 1, 2, 3, \ldots)$$
$$r^2 \qquad r \qquad 1$$

(5)

Solve the Auxillary Equation.

$r^2 = 6r - 9$

$\Rightarrow r^2 - 6r + 9 = 0$ (1)

$\Rightarrow (r-3)^2 = 0 \qquad \Rightarrow r = 3$ (1)

As we have a repeated root, we have

$U_n = (A + Bn)3^n$ (1)

Sub in initial conditions to make a pair of simultaneous.

$U_1 = 4 \Rightarrow 4 = (A+B)3$

$U_2 = 6 \Rightarrow 6 = (A+2B)9$ (1)

$\Rightarrow \left.\begin{array}{l} 4 = 3A + 3B \\ 6 = 9A + 18B \end{array}\right\} \begin{array}{l} A = 2 \\ B = -2/3 \end{array}$ using a calculator

Hence, $U_n = \left(2 - \frac{2}{3}n\right)3^n$ (1)

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3. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

(a) Use the Euclidean Algorithm to determine the highest common factor h of 234 and 96
(3)

(b) Hence determine integers a and b such that

$$234a + 96b = h$$
(3)

(c) Solve the congruence equation

$$96x \equiv 36 \pmod{234}$$
(5)

a) $234 = 2(96) + 42$ ①

$96 = 2(42) + 12$

$42 = 3(12) + 6$

$12 = 2(6) + 0$ ①

Hence, the highest common factor is 6. ①

b) We will sub in values we had before

$6 = 42 - 3(12)$ ①

$= 42 - 3(96 - 2(48))$

$= -3(96) + 7(42)$

$= -3(96) + 7(234 - 2(96))$ ①

$= 7(234) - 17(96)$

$\Rightarrow a = 7, \ b = -17$ ①

Question 3 continued

c) $96x \equiv 36 \pmod{234}$

as $\gcd(96, 234) = 6$

$\frac{96}{6}x = \frac{36}{6} \left(\bmod \frac{234}{6}\right)$

$\Rightarrow 16x = 6 \pmod{39}$ 1

We need to find $16^{-1} \bmod 39$

$16y \equiv 1 \pmod{39}$

using the Extended Euclidean Algorithm,

$39 = 2(16) + 7$

$16 = 2(7) + 2$

$7 = 3(2) + 1$

$2 = 2(1) + 0$

and working backwards,

$1 = 7 - 3(2)$

$= 7 - 3(16 - 2(7))$

$= 7(7) - 3(16)$ 1

Since $7 = 39 - 2(16)$,

Question 3 continued

$1 = (39 - 2(16))7 - 3(16)$

$= 7(39) - 14(16) - 3(16)$

$= 7(39) - 17(16)$

So $16^{-1} = -17 \equiv 22 \pmod{39}$ 1

Solve for x

$x = 6(22) \pmod{39}$

$\Rightarrow x = 132 \pmod{39}$

$132 \div 39 = 3$ r 15 1

so $x \equiv 15 \pmod{39}$ 1

is a solution.

4.

$$\mathbf{A} = \begin{pmatrix} 4 & 2 & 0 \\ 2 & p & -2 \\ 0 & -2 & 2 \end{pmatrix} \quad \text{where } p \text{ is a constant}$$

Given that $\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ is an eigenvector of **A**,

(a) determine the eigenvalue corresponding to this eigenvector.

(2)

(b) Hence show that $p = 3$

(1)

(c) Determine

(i) the remaining eigenvalues of **A**,

(ii) corresponding eigenvectors for these eigenvalues.

(6)

(d) Hence determine a matrix **P** and a diagonal matrix **D** such that $\mathbf{A} = \mathbf{PDP}^{\mathrm{T}}$

(3)

a) $\begin{pmatrix} 4 & 2 & 2 \\ 2 & p & -1 \\ 0 & -2 & 2 \end{pmatrix}\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -p \\ 6 \end{pmatrix} = \lambda \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ (1)

$\Rightarrow 6 = 2\lambda \Rightarrow \lambda = 3$ (1)

b) by part a,

$-p = -\lambda = -3 \Rightarrow p = 3$ (1)

c) Recall that $\det(A - \lambda I) = 0$

$\begin{vmatrix} 4-\lambda & 2 & 0 \\ 2 & 3-\lambda & -2 \\ 0 & -2 & 2-\lambda \end{vmatrix} = (4-\lambda)\begin{vmatrix} 3-\lambda & -2 \\ -2 & 2-\lambda \end{vmatrix} - 2\begin{vmatrix} 2 & -2 \\ 0 & 2-\lambda \end{vmatrix} = 0$ (1)

$\Rightarrow (4-\lambda)[(3-\lambda)(2-\lambda)-4] - 2(2(2-\lambda)) = 0$ (1)

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Question 4 continued

$\Rightarrow (4-\lambda)(2-5\lambda+\lambda^2) - 2(4-2\lambda) = 0$

$\Rightarrow 8 - 20\lambda + 4\lambda^2 - 2\lambda + 5\lambda^2 - \lambda^3 - 8 + 4\lambda = 0$

$\Rightarrow \lambda^3 - 9\lambda^2 + 18\lambda = 0$

$\Rightarrow \lambda(\lambda^2 - 9\lambda + 18) = 0$

$\Rightarrow \lambda(\lambda-3)(\lambda-6) = 0 \quad \Rightarrow \lambda = 0, 3 \text{ or } 6.$

So the remainding eigenvalues are 0 and 6 ①

ii) $\lambda = 0$:

$$\begin{pmatrix} 4 & 2 & 0 \\ 2 & 3 & -2 \\ 0 & -2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$4x + 2y = 0 \Rightarrow y = -2x$ ①

$-2y + 2z = 0 \Rightarrow z = -2x$

So an eigenvector is $\begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$ for $\lambda = 0$ ①

$\lambda = 6$:

$$\begin{pmatrix} -2 & 2 & 0 \\ 2 & -3 & -2 \\ 0 & -2 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$-2x + 2y = 0 \Rightarrow x = y$

$-2y - 4z = 0 \Rightarrow z = -\frac{1}{2}x$

Question 4 continued

So $\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ 1 is an eigenvector when $\lambda = 6$

d) $D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{pmatrix}$ 1 $\leftarrow$ eigenvalues on diagonals

$P = \frac{1}{v}(\underline{v_1}, \underline{v_2}, \underline{v_3})$ where $\underline{v_i}$ are eigenvectors

and $v = |\underline{v_i}|$

$|\underline{v}| = \sqrt{4+4+1} = 3$ 1

$P = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$ 1

5. (i) A circle C in the complex plane is defined by the locus of points satisfying

$$|z - 3\mathrm{i}| = 2|z|$$

(a) Determine a Cartesian equation for C, giving your answer in simplest form.

(3)

(b) On an Argand diagram, shade the region defined by

$$\{z \in \mathbb{C} : |z - 3\mathrm{i}| > 2|z|\}$$

(2)

(ii) The transformation T from the z-plane to the w-plane is given by

$$w = z^3$$

(a) Describe the geometric effect of T.

(2)

The region R in the z-plane is given by

$$\left\{z \in \mathbb{C} : 0 < \arg z < \frac{\pi}{4}\right\}$$

(b) On a **different** Argand diagram, sketch the image of R under T.

(2)

i) a) Let $z = x + iy$

$|x + iy - 3i| = 2|x + iy|$ ①

$\Rightarrow |x + i(y-3)| = 2|x + iy|$ $\qquad |x + iy| = \sqrt{x^2 + y^2}$

$\Rightarrow \sqrt{x^2 + (y-3)^2} = 2\sqrt{x^2 + y^2}$ ①

$\Rightarrow x^2 + (y-3)^2 = 4x^2 + 4y^2$

$\Rightarrow x^2 + y^2 - 6y + 9 = 4x^2 + 4y^2$

$\Rightarrow 3x^2 + 3y^2 + 6y - 9 = 0$

$\Rightarrow x^2 + y^2 + 2y - 3 = 0$ ①

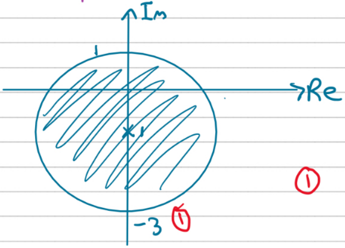
Question 5 continued

b) Complete the square to find the radius.

$x^2 + (y+1)^2 - 1 - 3 = 0$

$\Rightarrow x^2 + (y+1)^2 = 4$

which is a circle centred at $(0,-1)$ with a radius of 2



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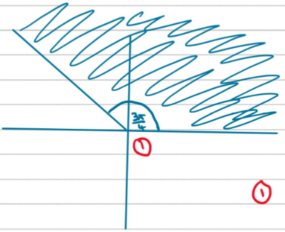
ii) a) A point z is mapped to a point with three times the arguement 1 and the modulus is the modulus of z cubed. 1

b)



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The arguement is $3 \times \frac{\pi}{4}$, and the modulus is not restricted, so is to infinity.

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6. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

$$I_n = \int \frac{\cos(nx)}{\sin x}\,dx \qquad n \geqslant 1$$

(a) Show that, for $n \geqslant 1$

$$I_{n+2} = \frac{2\cos(n+1)x}{n+1} + I_n$$

(6)

(b) Hence determine the exact value of

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\cos(5x)}{\sin x}\,dx$$

giving the answer in the form $a + b\ln c$ where a, b and c are rational numbers to be found.

(5)

a) $I_n = \int \frac{\cos(nx)}{\sin x}\,dx$

$\Rightarrow I_{n+2} = \int \frac{\cos(nx+2x)}{\sin x}\,dx$ 1

From the Formula Booklet, we know that

$\cos(A+B) = \cos A\cos B - \sin A \sin B$.

$\Rightarrow I_{n+2} = \int \frac{\cos(nx)\cos(2x) - \sin(nx)\sin(2x)}{\sin x}\,dx$ 1

Double angle rules

$\Rightarrow I_{n+2} = \int \frac{\cos(nx)(1-\sin^2 x) - 2\sin x\cos x\sin(nx)}{\sin x}\,dx$ 1

$\Rightarrow I_{n+2} = \int \frac{\cos(nx)}{\sin x} - \int \frac{\cos(nx)\sin^2 x - 2\sin x\cos x\sin(nx)}{\sin x}\,dx$

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Question 6 continued

$\Rightarrow I_{n+2} = I_n - 2\int \cos(nx)\sin(x) - \cos x \sin(nx)\, dx$ (1)

From the Formula Booklet, we know that

$\sin(A+B) = \sin(A)\cos(B) + \sin(B)\cos(A)$

$\Rightarrow I_{n+2} = I_n - 2\int \sin((n+1)x)\, dx$ (1)

$\Rightarrow I_{n+2} = I_n - 2\left[\frac{-\cos((n+1)x)}{n+1}\right]$

$\Rightarrow I_{n+2} = I_n + \frac{2\cos((n+1)x)}{n+1}$ (1)

b) The question asks for I_5, so we start with I_1 and work our way there.

$I_1 = \int_{\pi/4}^{\pi/3} \frac{\cos x}{\sin x}\, dx = \int_{\pi/4}^{\pi/3} \cot x\, dx$ (1) Rules of logs

$= \left[\ln \sin(x)\right]_{\pi/4}^{\pi/3} = \ln\frac{\sqrt{3}}{2} - \ln\frac{\sqrt{2}}{2} = \ln\frac{\sqrt{3}}{\sqrt{2}} = \frac{1}{2}\ln\frac{3}{2}$ (1)

$I_3 = \cos(2x) + I_1$ (1)

$I_5 = \frac{\cos(4x)}{2} + I_3$

$\Rightarrow I_5 = \left[\cos(2x) + \frac{\cos(4x)}{2}\right]_{\pi/4}^{\pi/3} + \frac{1}{2}\ln\frac{3}{2}$ (1)

$= \left[-\frac{1}{2} - \frac{1}{4} - 0 + \frac{1}{2}\right] + \frac{1}{2}\ln\frac{3}{2}$

$= \frac{1}{2}\ln\frac{3}{2} - \frac{1}{4}$ (1)

7. The set of matrices $G = \{\mathbf{I}, \mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{E}\}$ where

$$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \quad \mathbf{D} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \quad \mathbf{E} = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$$

with the operation $\otimes_2$ of matrix multiplication with entries evaluated modulo 2, forms a group.

(a) Show that $\mathbf{B}$ is an element of order 3 in G.

(2)

(b) Determine the orders of the other elements of G.

(3)

(c) Give a reason why G is **not** isomorphic to

(i) a cyclic group of order 6

(ii) the group of symmetries of a regular hexagon.

(2)

The group H of permutations of the numbers 1, 2 and 3 contains the following elements, denoted in two-line notation,

$$e = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \quad a = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \quad b = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

$$c = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \quad d = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \quad f = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$

(d) Determine an isomorphism between the groups G and H.

(3)

a) $B = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$

$\Rightarrow B^2 = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$

$\Rightarrow B^3 = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$ ①

hence, B has order 3. ①

Question 7 continued

b) I has order 1

$E = B^{-1}$ so has order 3 1

$A^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

So A has order 2 as we are working with modulo 2.

$C^2 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = I$

So C has order 2

$D = C^{-1}$, so D has order 2. 1

c) i) There is no element of order 6. 1

ii) There are 12 symmetries of a regular hexagon. 1

d) Find the order of all the elements in H.

e has order 1 because it is the identity permutation

d, c, f have order 2 because they swap two elements.

b and a have order 3 as they have three cycle permutations.

We can match all of the elements of H with an element of G.

Question 7 continued

G	I	A	B	C	D	E
H	e	d	a	c	f	b

(1) (1) (1)

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Figure 1

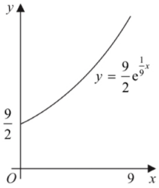

Figure 2

Figure 1 shows a French horn with a detachable bell section.

The shape of the bell section can be modelled by rotating an exponential curve through 360° about the x-axis, where units are centimetres.

The model uses the curve shown in Figure 2, with equation

$$y = \frac{9}{2}e^{\frac{1}{9}x} \qquad 0 \leqslant x \leqslant 9$$

(a) Show that, according to this model, the external surface area of the bell section is given by

$$K\int_0^9 e^{\frac{1}{9}x}\sqrt{4 + e^{\frac{2}{9}x}}\,dx$$

where K is a real constant to be determined.

(3)

(b) Use the substitution $u = e^{\frac{1}{9}x}$ to show that

$$\int_0^9 e^{\frac{1}{9}x}\sqrt{4 + e^{\frac{2}{9}x}}\,dx = 9\int_a^b \frac{2u + u^3}{\sqrt{4u^2 + u^4}}\,du + 18\int_a^b \frac{1}{\sqrt{4 + u^2}}\,du$$

where a and b are constants to be determined.

(5)

Hence, using algebraic integration,

(c) determine, according to the model, the external surface area of the bell section of the horn, giving your answer to 3 significant figures.

(5)

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Question 8 continued

a) Recall the formula

$$SA = 2\pi \int_a^b y\sqrt{1+\left(\frac{dy}{dx}\right)^2} \ dx$$

$$y = \frac{9}{2}e^{\frac{x}{9}} \Rightarrow \frac{dy}{dx} = \frac{1}{2}e^{x/9}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{1}{4}e^{\frac{2x}{9}} \quad (1)$$

$$SA = 2\pi \int_0^9 \frac{9}{2}e^{\frac{x}{9}} \sqrt{1+\frac{1}{4}e^{\frac{2x}{9}}} \ dx \quad (1)$$

$$\Rightarrow SA = \frac{9\pi}{2}\int_0^9 e^{x/9}\sqrt{4+e^{2x/9}} \ dx \quad (1)$$

b) Let $u = e^{x/9} \Rightarrow \frac{du}{dx} = \frac{1}{9}e^{x/9} \Rightarrow dx = 9e^{-x/9} du$ (1)

Change the limits

when $x = 0$, $u = 1$, when $x = 9$, $u = e$ (1)

So we have

$$\int_1^e u(4+u^2)^{1/2} \cdot 9u^{-1} \ du$$

$$= 9\int_1^e (4+u^2)^{1/2} \ du \quad (1)$$

$$= 9\int_1^e \frac{4+u^2}{(4+u^2)^{1/2}} \ du$$

Question 8 continued

$$= 9\int_1^e \frac{2+u^2}{(4+u^2)^{1/2}} + \frac{2}{(4+u^2)^{1/2}}\,du \quad 1$$

$$= 9\int_1^e \frac{2u+u^3}{u(4+u^2)^{1/2}}\,du + 18\int_1^e \frac{1}{(4+u^2)^{1/2}}\,du$$

$$= 9\int_1^e \frac{2u+u^3}{(4u^2+u^4)^{1/2}}\,du + 18\int_1^e \frac{1}{(4+u^2)^{1/2}}\,du \quad 1$$

c) Continue from part b, then multiply by k from part a.

$$= \frac{9}{4}\int_1^e \frac{8u+4u^3}{(4u^2+u^4)^{1/2}}\,du + 18\int_1^e \frac{1}{(4+u^2)^{1/2}}\,du \quad 1$$

Reverse Chain Rule

$$= \frac{9}{4}\left[2(4u^2+u^4)^{1/2}\right]_1^e + 18\left[\operatorname{arsinh}\left(\frac{u}{2}\right)\right]_1^e \quad 1 \quad 1$$

$$= \frac{9}{4}\left[2(4e^2+e^4)^{1/2} - 2(5)^{1/2}\right] + 18\left[\operatorname{arsinh}\left(\frac{e}{2}\right) - \operatorname{arsinh}\left(\frac{1}{2}\right)\right] \quad 1$$

$$= 42.6084.$$

$$SA = \frac{9\pi}{2} \times 42.6084 = 602\text{cm}^2. \quad 1$$

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